Generating genuine multipartite entanglement via XY-interaction and via von Neumann projections

Mazhar Ali
Theoretische Quantenoptik, Universität Siegen

QED^3, Bilbao

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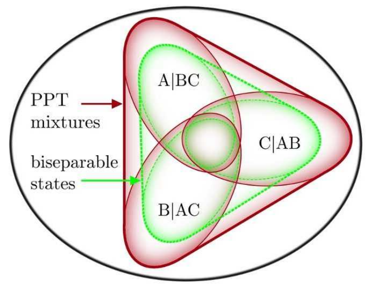
Outlines

- Motivations and applications
- Genuine multipartite entanglement (GME)
- Generating GME via XY-interaction
- Generating GME via von Neumann projections
- Conclusions

Multipartite entanglement and monotone

- Fully separable states: $|\psi^{fs}\rangle = |a\rangle |b\rangle |c\rangle$ & $\rho^{fs} = \sum_k p_k |\psi_k^{fs}\rangle\langle\psi_k^{fs}|$[1]
- Biseparable states: $|\psi^{bs}\rangle = |a\rangle |\psi^{bc}\rangle$ with $|\psi^{bc}\rangle$ possibly entangled and $\rho^{bs} = \sum_k p_k |\psi_k^{bs}\rangle\langle\psi_k^{bs}|$
- Genuinely multipartite entangled (GME): not of above types
- Entanglement monotone: Jungnitsch, Moroder, and Gühne, PRL. **106**, 190502



[1] Acin, Bruss, Lewenstein, and Sanpera, PRL. **87**, 040401

Generating GME via XY interactions

- Interaction Hamiltonian[2]

$$H^{ij}_{int} = \frac{\hbar g}{2}\left(\sigma^i_x\sigma^j_x + \sigma^i_y\sigma^j_y\right)$$

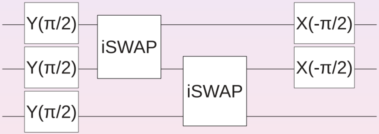
- Initial state 3 qubits: $|\psi(0)\rangle = \sum_{ijk=0}^{1} C_{ijk}(0)\,|ijk\rangle$
- Initial state 4 qubits: $|\phi(0)\rangle = \sum_{ijkl=0}^{1} C_{ijkl}(0)\,|ijkl\rangle$
- $C_{ij...}(0)$ are probability amplitudes & $|ij....\rangle$ computational basis
- Dynamics

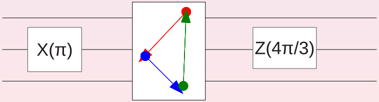
$$\mathrm{i}\hbar\frac{\partial}{\partial t}|\psi(t)\rangle = H_{int}|\psi(t)\rangle$$

[2] Neeley *et al*, Nature. **467**, 570 (2010).

Three qubit GME states

- $|GHZ_3\rangle = |000\rangle + |111\rangle$: $(E = 1/2)$
- *iSWAP* gate[3]: Applying H_{int}^{ij} for $t_{iSWAP} = \pi/(2g)$
- $|W_3\rangle = |001\rangle + |010\rangle + |100\rangle$: $(E = 0.4428)$
- Interaction Hamiltonian: $H_{int} = H_{int}^{AB} + H_{int}^{AC} + H_{int}^{BC}$ for time $t_W = (4/9)t_{iSWAP}$; $(gt_w \approx 0.7)$.

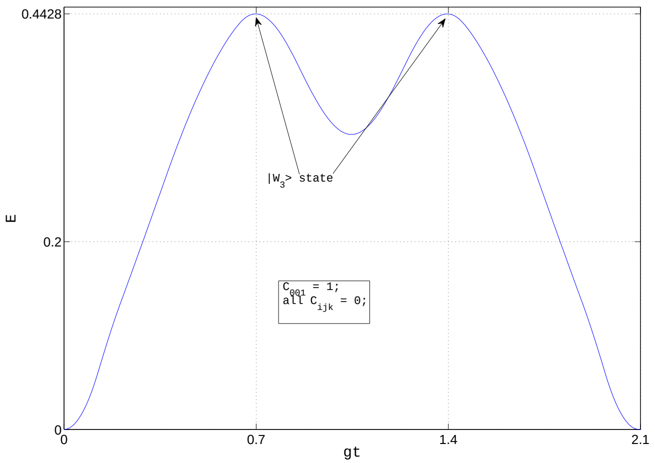




[3]Schuch and Siewert, Phys. Rev. A **67**, 032301

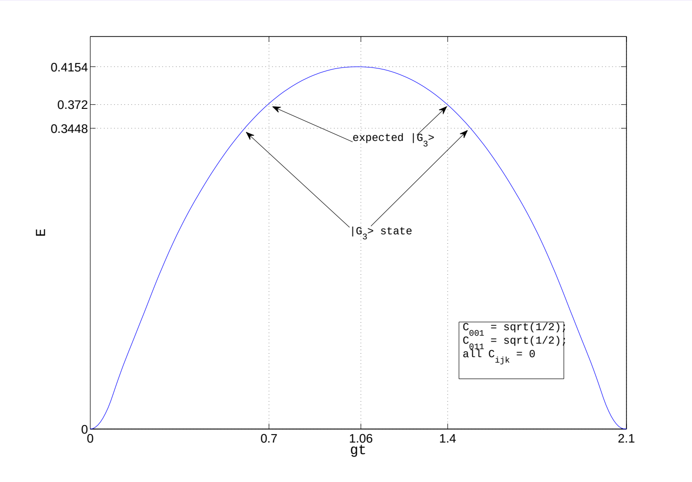
Generating $|W_3\rangle$ state

- Two set of coupled equations: $\{ C_{001}, C_{010}, C_{100} \}$ and $\{ C_{110}, C_{101}, C_{011} \}$



Generating $|G_3\rangle$ state

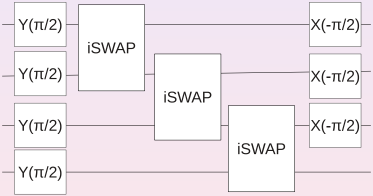
- Possibility of $|G_3\rangle$ states[4]: $|G_n\rangle = (|W_n\rangle + |\widetilde{W_n}\rangle)/\sqrt{2}$

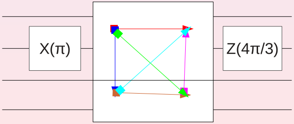


[4] Sen(De), Sen, and Zukowski, Phys. Rev. A 68, 032309

Generating GME for 4 qubits

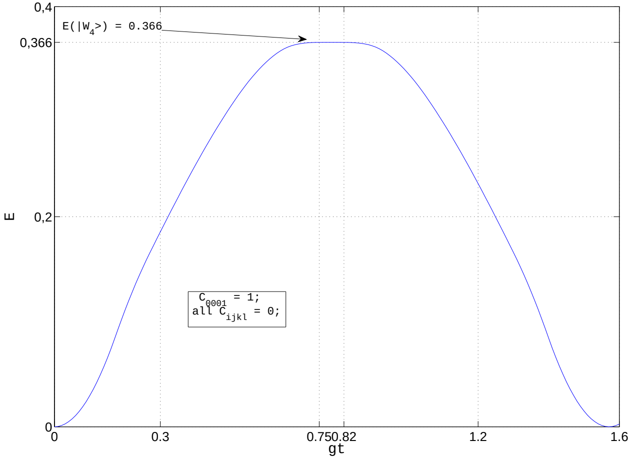
- $|GHZ\rangle = (|0000\rangle + |1111\rangle)$: $(E = 1/2)$
- Interaction Hamiltonian:
  $H_{int} = H_{int}^{AB} + H_{int}^{AC} + H_{int}^{AD} + H_{int}^{BC} + H_{int}^{BD} + H_{int}^{CD}$
- Three set of coupled equations: $\{\, C_{0001},\ C_{0010},\ C_{0100},\ C_{1000}\,\}$, $\{\, C_{1110},\ C_{1101},\ C_{1011},\ C_{0111}\,\}$, and $\{C_{0011}, \ldots C_{1100}\}$





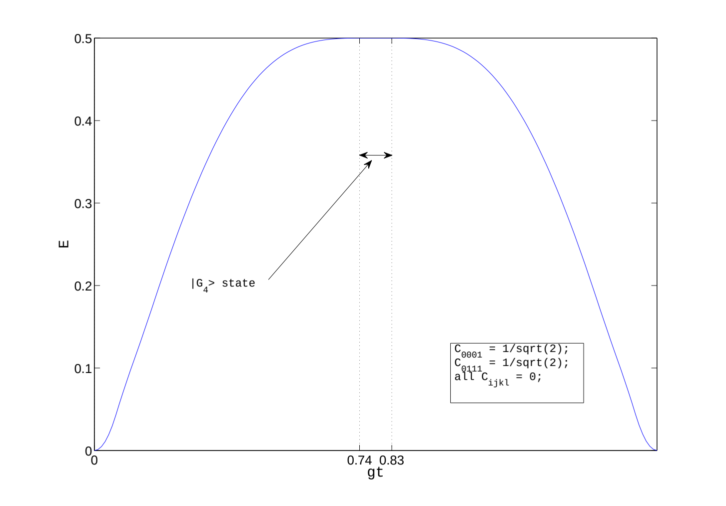
Generating $|W_4\rangle$ state

- $|W_4\rangle = |0001\rangle + |0010\rangle + |0100\rangle + |1000\rangle$: $(E = 0.366)$

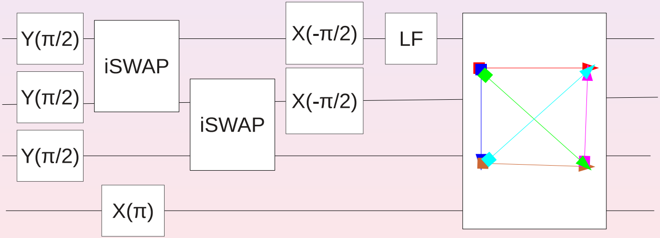


Generating $|G_4\rangle$ state

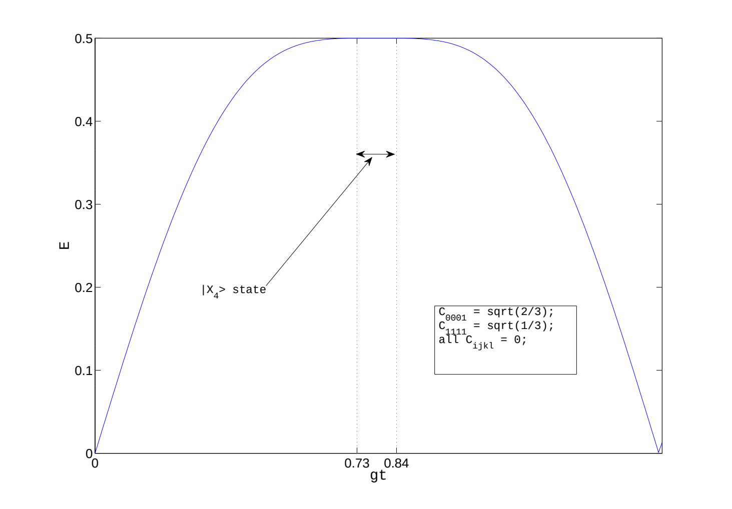
- $|G_4\rangle = 1/\sqrt{2}(|W_4\rangle + |\widetilde{W_4}\rangle)$: $E(|G_4\rangle) = 1/2$



- $|\chi_4\rangle = \sqrt{2}\,|1111\rangle + |0001\rangle + |0010\rangle + |0100\rangle + |1000\rangle$: $(E = 1/2)$
- First $|GHZ_3\rangle$: Local Filter: $f = diag\{a, 1/a\}$ with $a = (2)^{1/4}$
- $|\psi_{ABC}\rangle = \sqrt{2/3}\,|000\rangle + \sqrt{1/3}\,|111\rangle$
- Four body interaction (last qubit in excited state) for "t_w"

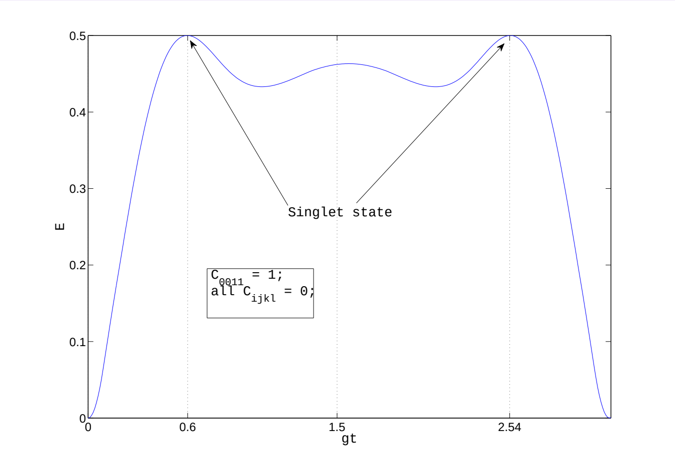


Generating $|\chi_4\rangle$ state



Generating Singlet state

- $|\Psi_{S,4}\rangle = |0011\rangle + |1100\rangle - 1/2(|0101\rangle + |0110\rangle + |1001\rangle + |1010\rangle$: $(E = 1/2)$



Generating GME via von Neumann projections

- Main idea: To get m-qubit state from n-qubit state for $n > m$ by projection operators[5]
- $\tilde{\rho}_m = Tr_{ij}[I\otimes \ldots M_i \otimes \ldots M_j \otimes \ldots I(\rho_n) I \otimes \ldots M_i^\dagger \otimes \ldots M_j^\dagger \otimes \ldots I]$:
- $M = V\Pi_i V^\dagger$, where $\Pi_i = |i\rangle\langle i|$ is projector and $V = tI + i\vec{y}\cdot\vec{\sigma}$
- $V \in SU(2)$ with $t^2 + y_1^2 + y_2^2 + y_3^2 = 1$, (3 parameters) and $t, y_i \in [-1, 1] \in \mathbb{R}$

[5] Kiesel *et al*, PRL. **98**, 063604; Wieczorek *et al*, PRL. **103**, 020504; PRA **79**, 022311

Results for $n = 4$ and $m = 3$

- $|GHZ_4\rangle \mapsto \alpha|000\rangle + \beta|111\rangle$: where $\alpha, \beta := f(t, y_i)$
- $|W_4\rangle \mapsto \alpha|W_3\rangle + \beta|000\rangle$
- $|CL_4\rangle \mapsto \sum_k \alpha_k |GHZ_3^k\rangle$
- $|D_{2,4}\rangle \mapsto \alpha|W_3\rangle + \beta|\widetilde{W}_3\rangle \mapsto |G_3\rangle$
- $|PS_4\rangle \mapsto \alpha|W_3\rangle + \beta|\widetilde{W}_3\rangle \mapsto |G_3\rangle$
- $|\chi_4\rangle \mapsto \alpha|W_3\rangle + \beta|\widetilde{GHZ}_3\rangle$
- $\widetilde{GHZ}_3 = \sqrt{1/3}|000\rangle + \sqrt{2/3}|111\rangle \mapsto^{LF} |GHZ_3\rangle$

Conclusions

- Generation of multipartite entangled states for 3 and 4 qubits - superconducting phase qubits
- Model has the capacity to generate all well known GME states
- Von Neumann projection to generate GME states in a lower dimensional space
- Better understanding - structure of GME states

Thanks for your attention